Midterm 2

for Math 308, Winter 2017

NAME (last - first): ____

- Do not open this exam until you are told to begin. You will have 50 minutes for the exam.
- This exam contains 5 questions for a total of 50 points in 9 pages.
- You are allowed to have one double sided, handwritten note sheet and a non-programmable calculator.
- Show all your work. With the exception of True/False questions, if there is no work supporting an answer (even if correct) you will not receive full credit for the problem.

Do not write on this table!

Question	Points	Score
1	6	
2	4	
3	14	
4	12	
5	14	
Total:	50	

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ____ Date: ____________________

Question 1. (6 points) Decide whether the following statements are true or false. For this you don't need to show any work.

(a) [1 point] If v is an eigenvector of A with eigenvalue λ then $v \in E_\lambda(A)$.

○ True ○ False

(b) [1 point] If λ is an eigenevalue of A then $\text{nullity}(A - \lambda I) \neq 0$.

○ True ○ False

(c) [1 point] If A is a $m \times n$ matrix, $\text{col}(A)$ is a subspace of $\mathbb{R}^m$.

○ True ○ False

(d) [1 point] If A is a $n \times n$ matrix with $\det(A) = 0$ then $\text{rank}(A) = n$.

○ True ○ False

(e) [1 point] The set $\{(a, b, c, d) \in \mathbb{R}^4 : a - b = cd^2\}$ is a subspace of $\mathbb{R}^4$.

○ True ○ False

(f) [1 point] If S is a subspace, $\mathcal{B}$ is a basis of S, and $u, v \in \mathcal{B}$ then $u + v \in \mathcal{B}$.

○ True ○ False

Question 2. (4 points) For any of the following question, give an explicit example.

(a) [1 point] A 3×3 matrix with nullity $= 2$.

(b) [1 point] A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ which is 1-to-1 but not onto.

(c) [1 point] A subspace S in $\mathbb{R}^3$ of dimension 2.

(d) [1 points] A 2×2 matrix with eigenvalues 0 and 1.

Question 3. (14 points) Let A be the following matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

(a) [2 points] Without doing any computation, what is $\det(A)$? Is A invertible? Why?

(b) [3 points] Find all the eigenvalues of A and their multiplicities.

(c) [6 points] Compute bases for all the eigenspaces of A.

(d) [1 points] Does there exists a basis of $\mathbb{R}^3$ made of eigenvectors of A? Explain.

(e) [2 point] Find a vector $v \in \mathbb{R}^3$ of length 5 such that $A \cdot v = 2v$. (recall that the length of a vector $u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ is $\|u\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$).

Question 4. (12 points) Given a 5×5 matrix A we know that it has the following eigenspaces (where the vectors form a basis of each eigenspace):

$$E_1(A) = \text{span}\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix} \quad E_2(A) = \text{span}\begin{pmatrix}0\\1\\1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\0\\0\\1\\0\end{pmatrix} \quad E_0(A) = \text{span}\begin{pmatrix}1\\0\\0\\-1\\0\end{pmatrix} \quad E_{-1}(A) = \text{span}\begin{pmatrix}0\\1\\2\\2\\2\end{pmatrix}$$

CAUTION: the vectors above are NOT the columns of A.

(a) [2 points] Write down a basis for the null space of A. Explain. (hint: what is the relation between the null space and one of the eigenspaces).

(b) [2 points] Is T_A 1-to-1, onto and/or invertible?

(c) [3 points] Without doing any computation, what is the rank of $A - 2I_5$? [Hint: what is the null space of that matrix?]

(d) [1 points] Does there exists a basis of $\mathbb{R}^5$ of eigenvectors of A?

(e) [4 points] Find all the vectors $x \in \mathbb{R}^5$ such that $T_A(x) = -x$. [Hint: try to rewrite the set of vectors as the null space of a certain matrix and relate that to the eigenspaces.]

Question 5. (14 points) Consider the following matrix

$$A = \begin{pmatrix} 1 & 0 & 2 & 1 & 0 & 1 \\ 1 & -1 & -2 & -1 & 1 & 2 \\ 2 & -2 & -4 & -2 & 1 & 2 \\ -1 & 1 & 2 & 1 & 1 & 2 \end{pmatrix}$$

(a) [6 points] Compute bases for row(A) and col(A).

(b) [2 points] What is rank(A)? What is nullity(A)?

(c) [3 points] Denoted by T_A the linear transformation associated to A (i.e. $T_A(x) = A \cdot x$), is T_A 1-to-1? Is it onto? Is it invertible?

(d) [1 points] How many elements will a basis of null(A) contain? (you don't need to do any computations for this)

(e) [2 points] Compute a basis for null(A) (hint: use the computation you already perform in part 1).